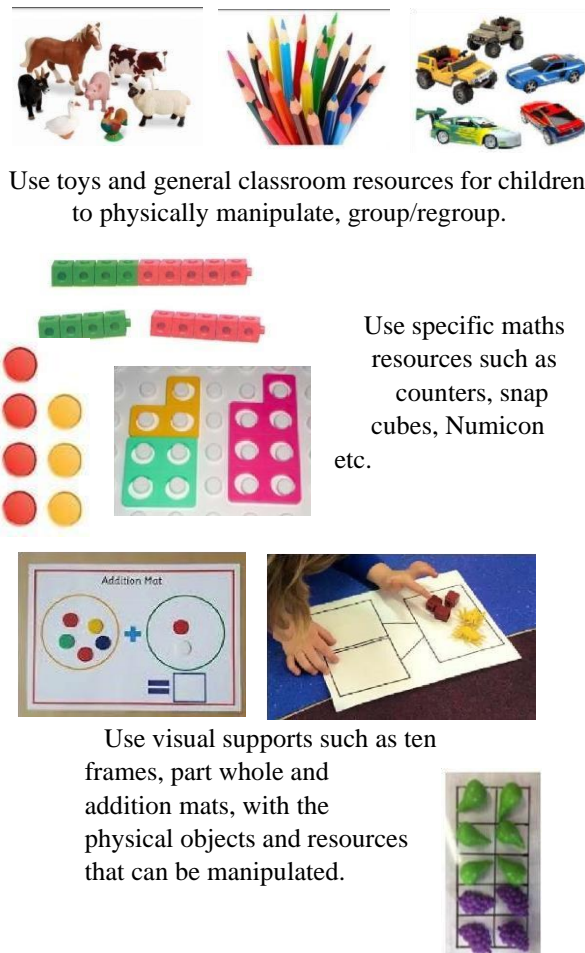
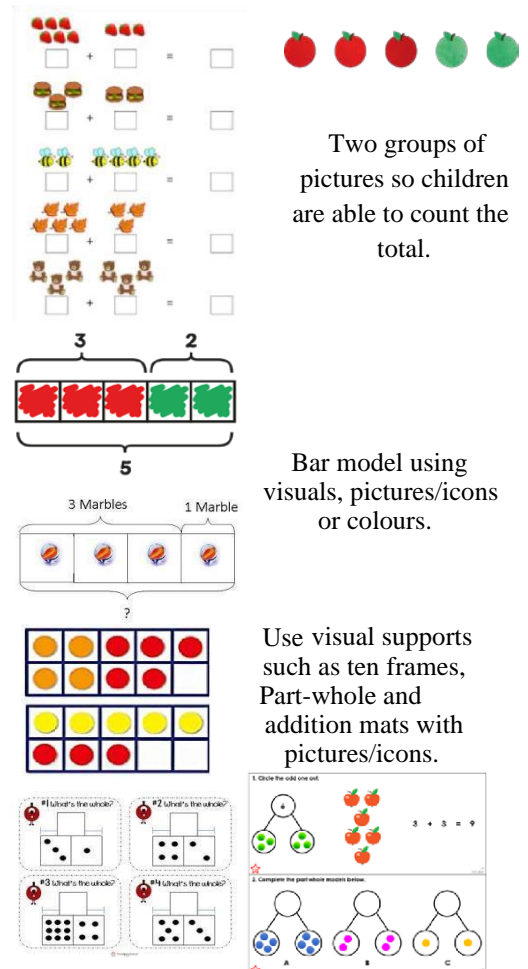
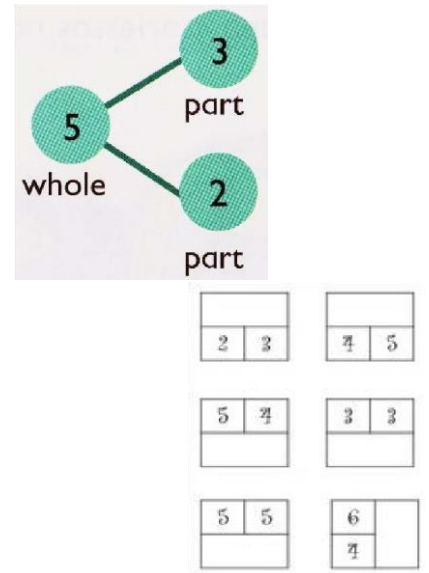

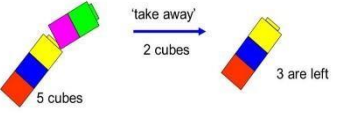
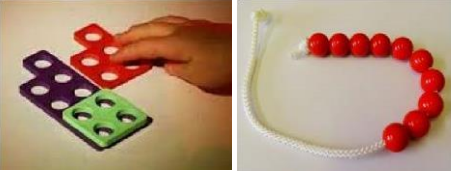
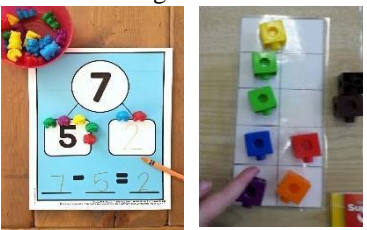
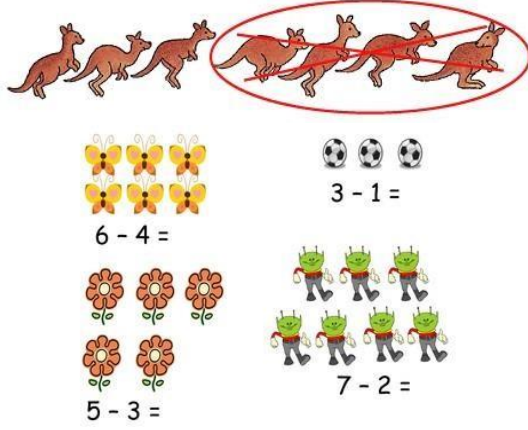
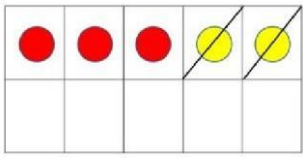
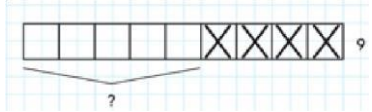
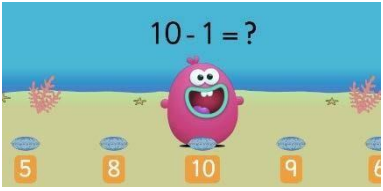
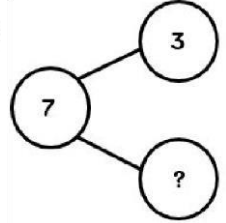


Addition- EYFS

Objectives	Concrete	Pictorial	Abstract
<p>Knows that a group of things change in quantity when something is added.</p> <p>Find the total number of items in two groups by counting all of them.</p> <p>Says the number that is one more than a given number.</p> <p>Finds one more from a group of up to five objects, then ten objects.</p> <p>In practical activities and discussion, beginning to use the vocabulary involved in adding.</p> <p>Using quantities and objects, they add two single digit numbers and count on to find the answer.</p>	<p style="text-align: center;">Concrete</p>  <p style="text-align: center;">Use toys and general classroom resources for children to physically manipulate, group/regroup.</p> <p style="text-align: center;">Use specific maths resources such as counters, snap cubes, Numicon etc.</p> <p style="text-align: center;">Use visual supports such as ten frames, part whole and addition mats, with the physical objects and resources that can be manipulated.</p>	<p style="text-align: center;">Pictorial</p>  <p style="text-align: center;">Two groups of pictures so children are able to count the total.</p> <p style="text-align: center;">Bar model using visuals, pictures/icons or colours.</p> <p style="text-align: center;">Use visual supports such as ten frames, Part-whole and addition mats with pictures/icons.</p>	<p style="text-align: center;">Abstract</p> <p>A focus on symbols and numbers to form a calculation.</p> <div style="border: 1px solid black; padding: 5px; display: inline-block; font-size: 2em; margin: 10px;"> $5 + 2 = 7$ </div>  <p style="text-align: center;">No expectation for children to be able to record a number sentence/addition calculation.</p>

Subtraction- EYFS

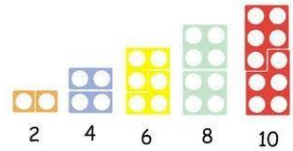
Objectives	Concrete	Pictorial	Abstract				
<p>Knows that a group of things change in quantity when something is taken away</p> <p>Find one less from a group of five objects, then ten objects.</p> <p>In practical activities and discussion, beginning to use the vocabulary involved in subtracting.</p> <p>Using quantities and objects, they subtract two single digit numbers and count back to find the answer.</p>	<div style="text-align: center;">  <p>Use toys and general classroom resources for children to physically manipulate, group/regroup.</p>   <p>Use specific maths resources such as snap cubes, Numicon, bead strings etc.</p>  <p>Use visual supports such as ten frames, part-whole and subtraction mats, with the physical objects and resources that can be manipulated.</p> </div>	<div style="text-align: center;">  <p>Use visual supports such as ten frames, part-whole and bar model with pictures/icons.</p>   </div>	<p>A focus on symbols and numbers to form a calculation.</p> <div style="text-align: center;">  <div style="border: 2px solid yellow; padding: 5px; display: inline-block; margin: 10px 0;"> $10 - 6 = 4$ </div> <table border="1" style="margin: 10px auto; border-collapse: collapse;"> <tr> <td style="padding: 5px;">3</td> <td style="padding: 5px;">?</td> </tr> <tr> <td colspan="2" style="text-align: center; padding: 5px;">7</td> </tr> </table> <p style="text-align: center; margin: 10px 0;">$7 - 3 = ?$</p> <div style="text-align: center;">  </div> <p style="text-align: center; margin-top: 20px;">* No expectation for children to be able to record a number sentence/addition calculation.</p> </div>	3	?	7	
3	?						
7							

Multiplication-EYFS

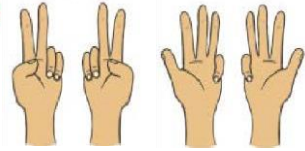
Objectives

Solve problems including doubling

Concrete

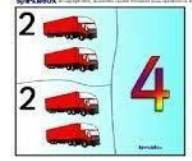
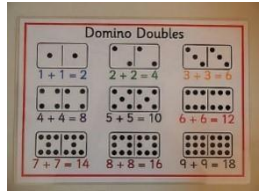
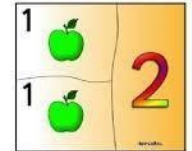
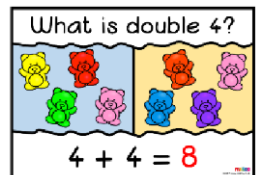


Counting and other maths resources for children to make 2 equal groups.



Physical and real life examples that encourage children to see the concept of doubling as adding two groups.

Pictorial



Pictures and icons that encourage children to see concept of doubling as adding two equal groups.

Abstract

1+1=	7+7=
2+2=	8+8=
3+3=	9+9=
4+4=	10+10=
5+5=	11+11=
6+6=	12+12=

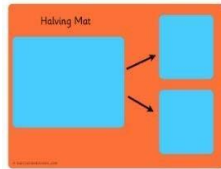
Addition calculations to model adding two equal groups.

Division- EYFS

Objectives	Concrete	Pictorial	Abstract
<p>Solve problems including halving and sharing.</p> <p>Halving a whole, halving a quantity of objects.</p> <p>Sharing a quantity of objects.</p>	<p>Children have the opportunity to physically cut objects, food or shapes in half.</p> <p>Use visual supports such as halving mats and part-whole with the physical objects and resources that can be manipulated</p> <p>Counting and other maths resources for children to explore sharing between 3 or more</p>	<p>Pictures and icons that encourage children to see concept of halving in relation to subitising, addition and subtraction knowledge. i.e. Knowing 4 is made of 2 groups of 2, so half of 4 is 2.</p> <p>Bar model with pictures or icons to support understanding of finding 2 equal parts of a number, to further understand how two halves make a whole.</p> <p>Pictures for children to create and visualise 3 or more.</p>	



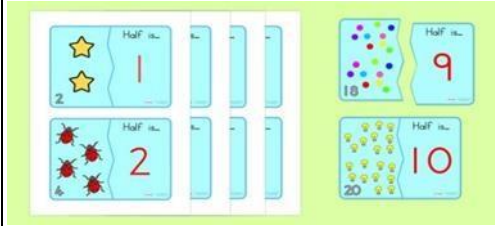
Children have the opportunity to physically cut objects, food or shapes in half.



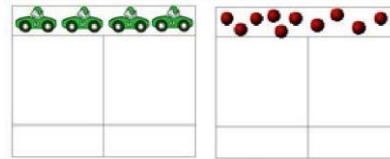
Use visual supports such as halving mats and part-whole with the physical objects and resources that can be manipulated



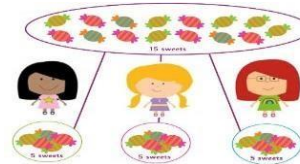
Counting and other maths resources for children to explore sharing between 3 or more



Pictures and icons that encourage children to see concept of halving in relation to subitising, addition and subtraction knowledge. i.e. Knowing 4 is made of 2 groups of 2, so half of 4 is 2.

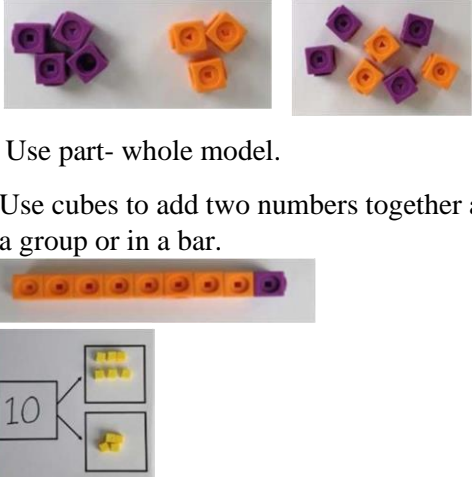
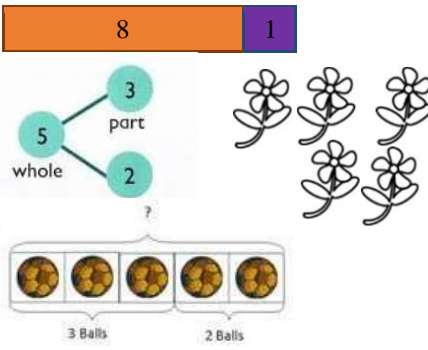


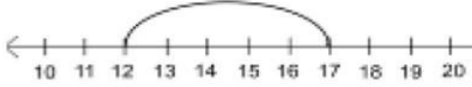


Bar model with pictures or icons to support understanding of finding 2 equal parts of a number, to further understand how two halves make a whole.

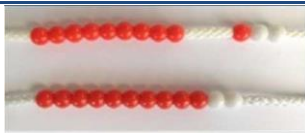


Pictures for children to create and visualise 3 or more.

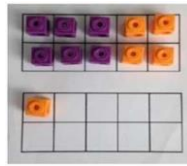
YEAR 1 Addition

Objective / Strategy	Concrete	Pictorial	Abstract
<p>Combining two parts to make a whole: part- whole model</p>	 <p>Use part- whole model.</p> <p>Use cubes to add two numbers together as a group or in a bar.</p>	<p>Use pictures to add two numbers together as a group or in a bar.</p> 	<p>$8 = 5 + 3$ 5</p> <p>+ 3 = 8</p>  <p>Use the part- whole diagram as shown above to move into the abstract.</p> <p>Include missing number questions to support varied fluency:</p> <p>$8 = ? + 3$</p> <p>$5 + ? = 8$</p>
<p>Starting at the bigger number and counting on</p>	 <p>Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.</p>	 <p>$12 + 5 = 17$</p> <p>Start at the larger number on the number line and count on in ones or in one jump to find the answer.</p>	<p>$5 + 12 = 17$</p> <p>Place the larger number in your head and count on the smaller number to find your answer.</p>

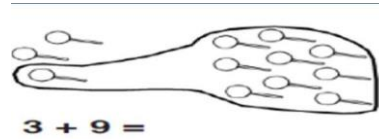
Regrouping to make 10.



$$6 + 5 = 11$$

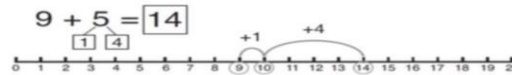


Start with the bigger number and use the smaller number to make 10. Use ten frames.



$$3 + 9 =$$

Use pictures or a number line. Regroup or partition the smaller number using the part, part whole model to make 10.



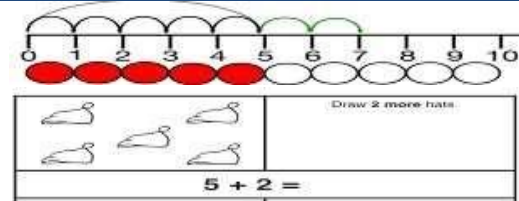
$$7 + 4 = 11$$

If I am at seven, how many more do I need to make 10? How many more do I add on now?

Represent & use number bonds and related subtraction facts within 20



2 more than 5.



Include missing number questions:

$$8 = ? + 3$$

$$5 + ? = 8$$

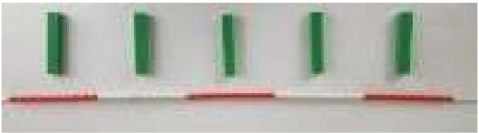
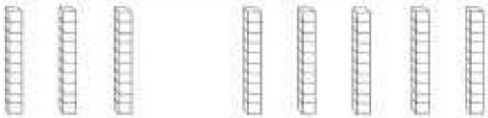
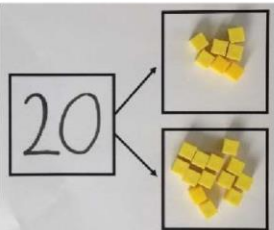
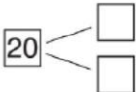
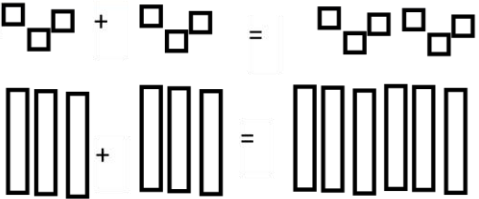
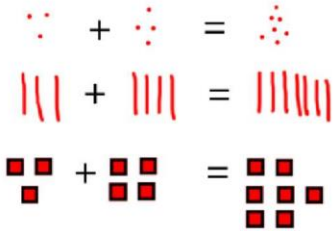
Emphasis should be on the language



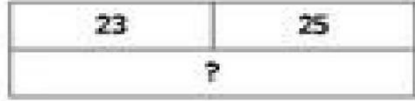
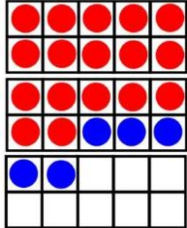
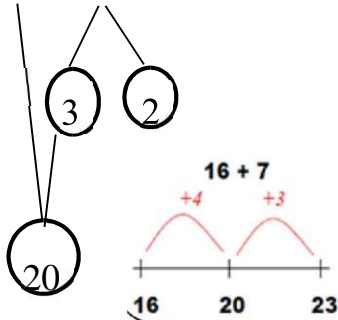
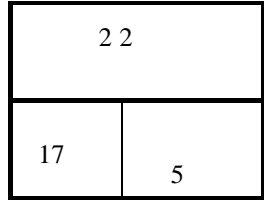

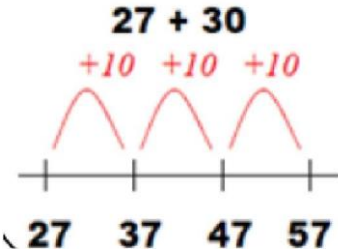
'1 more than 5 is equal to 6.'


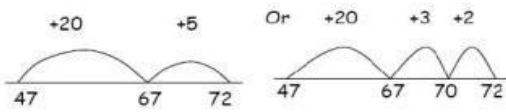

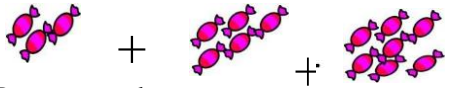

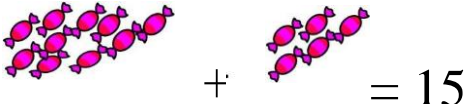
'2 more than 5 is 7.'

'8 is 3 more than 5.'

YEAR 2 Addition

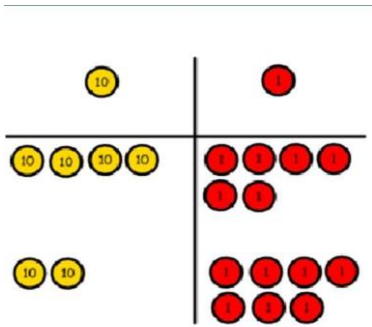
Objective /Strategy	Concrete	Pictorial	Abstract
Adding multiples of ten	<p style="text-align: center;">$50 = 30 + 20$</p>  <p style="text-align: center;">Model using dienes and bead strings</p>	 <p style="text-align: center;">3 tens + 5 tens = _____ tens $30 + 50 = \underline{\quad}$</p> <p style="text-align: center;">Use representations for base ten.</p>	$20 + 30 = 50$ $70 = 50 + 20$ $40 + \square = 60$
Use known number facts Part, part whole	 <p style="text-align: center;">Children explore ways of making numbers within 20</p>	 $\square + \square = 20$ $20 - \square = \square$ $\square + \square = 20$ $20 - \square = \square$	Explore commutativity of addition by swapping the addends to build a fact family. Explore the concept of the inverse relationship of addition and subtractions and use this to check calculations.
Using known facts		 <p style="text-align: center;">Children draw representations of H,T and O</p>	<div style="border: 1px solid gray; padding: 5px; margin: 5px;"> $3 + 4 = 7$ <i>leads to</i> $30 + 40 = 70$ <i>leads to</i> $300 + 400 = 700$ </div>

<p>Bar model</p>	 $3 + 4 = 7$	 $7 + 3 = 10$	 $23 + 25 = 48$
<p>Add a two-digit number and ones</p>	 <p>$17 + 5 = 22$</p> <p>Use ten frame to make 'magic ten.'</p> <p>Children explore the pattern.</p> $17 + 5 = 22$ $27 + 5 = 32$	<p>Use part-whole and number line to model.</p> <p>$17 + 5 = 22$</p> 	<p>$17 + 5 = 22$</p> <p>Explore related facts.</p> $17 + 5 = 22$ $5 + 17 = 22$ $22 - 17 = 5$ $22 - 5 = 17$  <p>Lead into recording in column format, to reinforce place value and prepare children for formal written methods with larger values.</p>
<p>Add a 2-digit number and tens</p>	 $25 + 10 = 35$ <p>Explore that the ones digit does not change</p>	<p>$27 + 30$</p> 	$27 + 10 = 37$ $27 + 20 = 47$ $27 + \square = 57$

<p>Add two 2-digit numbers</p>	 <p>Model using dienes, place value counters and numicon</p>	 <p>Use number line and bridge ten using part whole if necessary.</p>	$\begin{array}{r} \swarrow \quad \downarrow \quad \searrow \\ 25 + 47 \\ \downarrow \quad \downarrow \\ 20 + 5 \quad 40 + 7 \\ 20 + 40 = 60 \\ 5 + 7 = 12 \\ 60 + 12 = 72 \end{array}$
			<p>Lead into recording in column format, to reinforce place value and prepare children for formal written methods with larger values.</p>
<p>Add three 1-digit numbers</p>	 <p>Combine to make 10 first if possible, or bridge 10 then add third digit</p>	 <p>Regroup and representation.</p>  	$\begin{array}{r} (4) + 7 + (6) = \boxed{10} + \boxed{7} \\ \underbrace{\hspace{2cm}}_{10} \\ = \boxed{17} \end{array}$ <p>Combine the two numbers that make/ bridge ten then add on the third.</p>

YEAR 3 Addition

Objective /Strategy	Concrete	Pictorial	Abstract
<p>Column Addition—no regrouping (friendly numbers)</p> <p>Add two or three 2 or 3digit numbers.</p>	<p>Dienes or numicon</p> <p>Add together the ones first, then the tens.</p> <p>7 9</p> <p>Calculations $21 + 42 =$ 21 $+ 42$</p>	<p>Children move to drawing the counters using a tens and one frame.</p> <p>tens ones</p>	$\begin{array}{r} 223 \\ + 114 \\ \hline 337 \end{array}$ <p>Add the ones first, then the tens, then the hundreds.</p>
<p>Column Addition with regrouping.</p>	<p>Tens Units</p> <p>39 15</p> <p>5 4</p> <p>Exchange ten ones for a ten. Model using numicon and place value counters.</p>	$\begin{array}{r} 34 \\ + 17 \\ \hline \end{array}$ <p>Children can draw a representation of the grid to further support their understanding, carrying the ten underneath the line.</p>	$\begin{array}{r} 20 + 5 \\ 40 + 8 \\ 60 + 13 = 73 \end{array}$ <p>Start by partitioning the numbers before formal column to show the exchange.</p> $\begin{array}{r} 536 \\ + 85 \\ \hline 621 \\ 11 \end{array}$



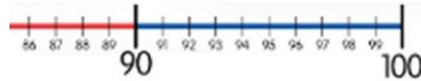
$$46 + 27 = 73$$

Estimate the answers to questions and use inverse operations to check answers



Estimating $98 + 17 = ?$
 $100 + 20 = 120$

Use number lines to illustrate estimation.



Building up known facts and using them to illustrate the inverse and to check answers:

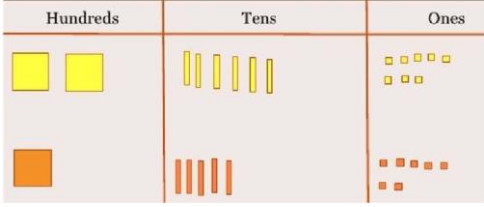
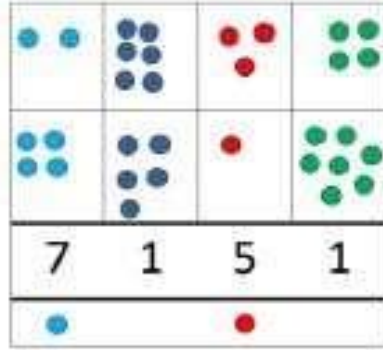
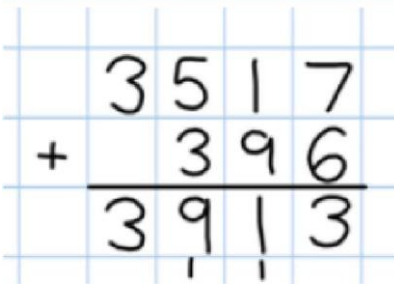
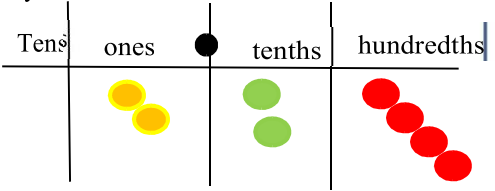
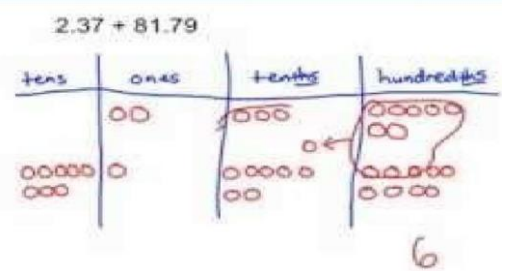
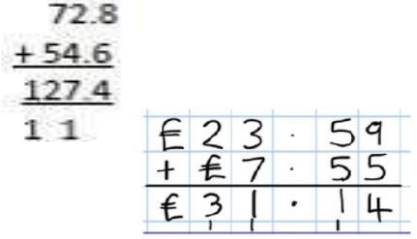
$$98 + 18 = 116$$

$$116 - 18 = 98$$

$$18 + 98 = 116$$

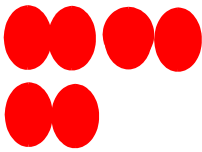

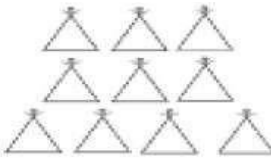
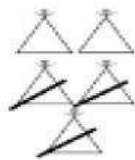
$$116 - 98 = 18$$

YEARS 4 – 6 Addition

Objective /Strategy	Concrete	Pictorial	Abstract
<p>Years 4 – 6</p> <p>Estimate and use inverse operations to check answers to a calculation</p>	<p>AS per Year 3</p>		
<p>Y4—add numbers with up to 4 digits</p>	<p>Children continue to use dienes or place value counters to add, exchanging ten ones for a ten and ten tens for a hundred and ten hundreds for a thousand.</p> 	 <p>Draw representations using place value grid.</p>	 <p>Continue from previous work to carry hundreds as well as tens.</p> <p>Relate to money and measures.</p>
<p>Y5—add numbers with more than 4 digits.</p> <p>Add decimals with 2 decimal places, including money.</p>	<p>As year 4</p>  <p>Introduce decimal place value counters and model exchange for addition.</p>		

<p>Y6—add several numbers of increasing complexity, including adding money, measure and decimals with different numbers of decimal points.</p>	<p>As Y5</p>	<p>As Y5</p>	<p>Insert zeros for place holders.</p> <div style="display: flex; justify-content: space-around; align-items: flex-start;"> <div style="text-align: center;"> $\begin{array}{r} 81,059 \\ + 3,668 \\ \hline 15,301 \\ + 20,551 \\ \hline 120,579 \\ \hline 1111 \end{array}$ </div> <div style="text-align: center;"> $\begin{array}{r} 23.361 \\ 9.080 \\ 59.770 \\ + 1.300 \\ \hline 93.511 \\ \hline 21211 \end{array}$ </div> </div>
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YEAR 1 SUBTRACTION

Objective /Strategy	Concrete	Pictorial	Abstract
<p>Taking away ones.</p>	<p>Use physical objects, counters, cubes etc to show how objects can be taken away.</p> <p>$4 - 2 = 2$</p>  <p>$6 - 4 = 2$</p> 	<p>Cross out drawn objects to show what has been taken away.</p>  <p>$15 - 3 = \boxed{12}$</p> 	<p>$7 - 4 = 3$</p> <p>$16 - 9 = 7$</p>

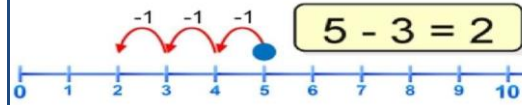
Counting back



Move objects away from the group, counting backwards.



Move the beads along the bead string as you count backwards.

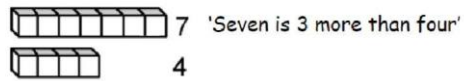


Count back in ones using a number line.

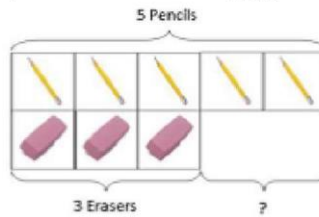
Put 13 in your head, count back 4. What number are you at?

Find the Difference

Compare objects and amounts

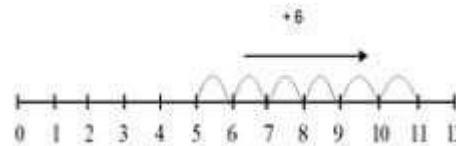


'I am 2 years older than my sister'

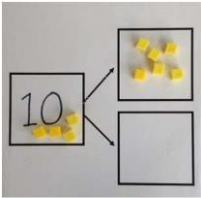
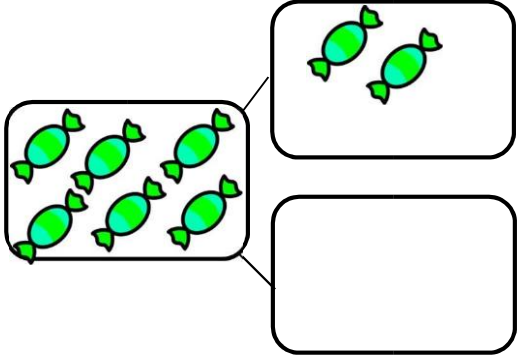
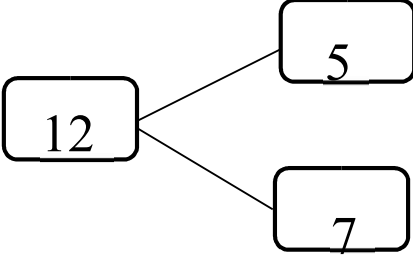
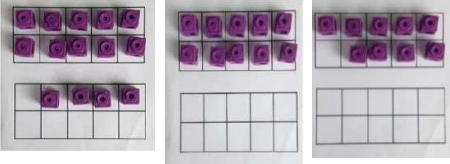
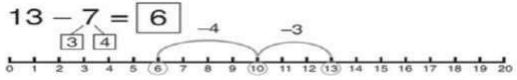


Lay objects to represent bar model.

Count on using a number line to find the difference.

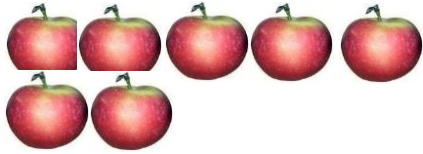


Hannah has 12 sweets and her sister has 5. How many more does Hannah have than her sister.?

Objective/Strategy	Concrete	Pictorial	Abstract
<p>Represent and use number bonds and related subtraction facts within 20</p> <p>Include subtracting zero</p> <p>Part- Whole model</p>	 <p>Link to addition. Use part-whole model to link to the inverse.</p> <p>If 10 is the whole and 6 is one of the parts, what is the other part?</p> $10 - 6 = 4$	 <p>Use pictorial representations to show the part.</p>	<p>Move to using numbers within the part-whole model.</p>  <p>Include missing number problems: $12 - ? = 5$ $7 = 12 - ?$</p>
<p>Make 10</p>	<p>14—9</p>  <p>Make 14 on the ten frame. Take 4 away to make ten, then take one more away so that you have taken 5.</p>	<p>$13 - 7 = \boxed{6}$</p>  <p>13—7</p> <p>Jump back 3 first, then another 4. Use ten as the stopping point.</p>	<p>16—8</p> <p>How many do we take off first to get to 10?</p> <p>How many left to take off?</p>

Bar model

Including the
inverse operations.



$$5 - 2 = 3$$



8	2
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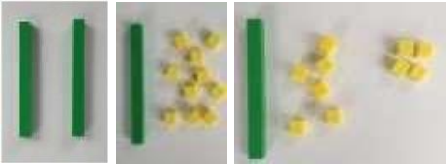

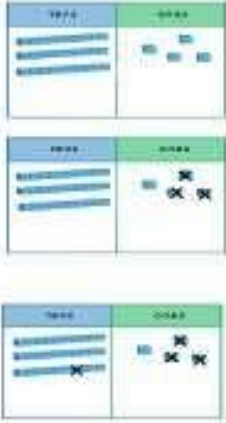
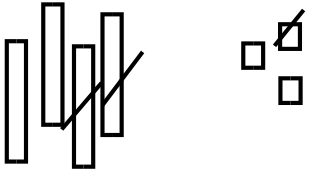
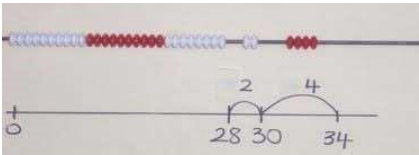
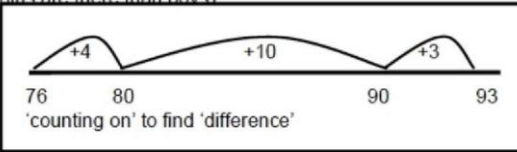
$$10 = 8 + 2$$

$$10 = 2 + 8$$



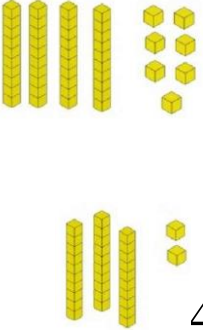
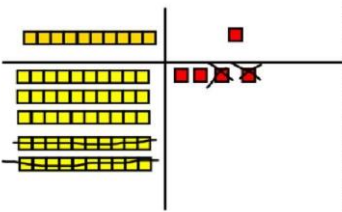
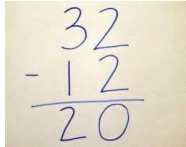
$$10 - 2 = 8$$

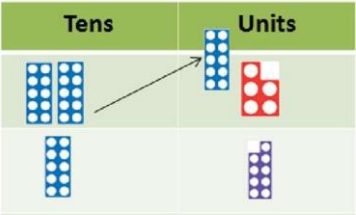
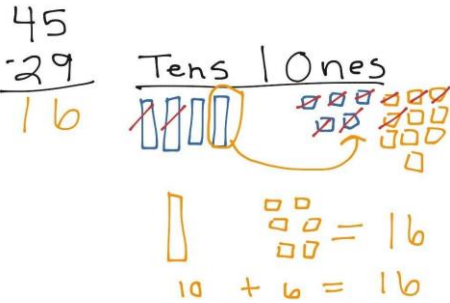
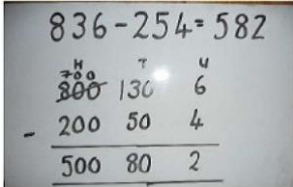
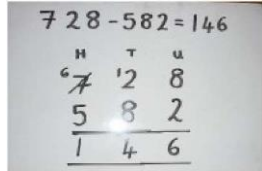
$$10 - 8 = 2$$

YEAR 2 - SUBTRACTION

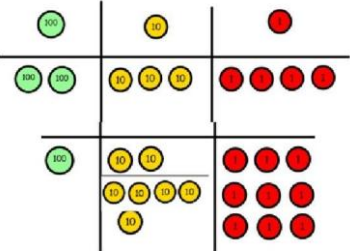
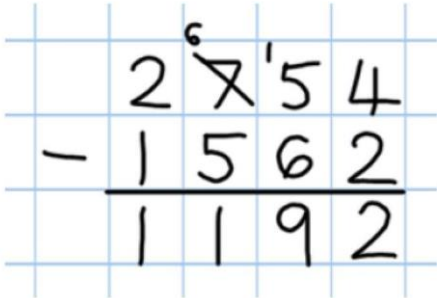
Objective & Strategy	Concrete	Pictorial	Abstract
<p>Regroup a ten into ten ones</p>	 <p>Use a PV chart to show how to change a ten into ten ones, use the term 'take and make'</p>	 <p>$20 - 4 = 16$</p>	<p>$20 - 4 = 16$</p>
<p>Partitioning to subtract without regrouping. 'Friendly numbers'</p>	<p>$34 - 13 = 21$</p>  <p>Use Dienes to show how to partition the number when subtracting without regrouping.</p>	<p>Children draw representations of Dienes and cross off.</p>  <p>$43 - 21 = 22$</p>	<p>$43 - 21 = 22$</p>
<p>Make ten strategy Progression should be crossing one ten, crossing more than one ten, crossing the hundreds.</p>	 <p>$34 - 28$</p> <p>Use a bead bar or bead strings to model counting to next ten and the rest.</p>	 <p>Use a number line to count on to next ten and then the rest.</p>	<p>$93 - 76 = 17$</p>

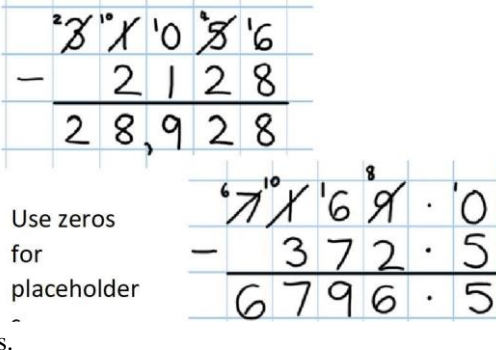

YEAR 3 - SUBTRACTION

Objective/ Strategy	Concrete	Pictorial	Abstract
<p>Subtract numbers mentally, including:</p> <ul style="list-style-type: none"> three digit number + ones three digit number + tens three digit number + hundreds 			<p>Vary the position of the answer and question.</p> <p>Expose children to missing number questions and vary the missing part of the calculation.</p> $678 = ? - 1$ $688 - 10 = ?$ $678 = ? - 100$
<p>Column subtraction without regrouping (friendly numbers)</p>	 <p style="text-align: center;">$47 - 32$</p> <p>Use base 10 or Numicon to model</p>	 <p style="text-align: right;">Calculations</p> $\begin{array}{r} 54 \\ - 22 \\ \hline 32 \end{array}$ <p>Draw representations to support understanding</p>	$47 - 24 = 23$ $\begin{array}{r} 40 + 7 \\ - 20 + 4 \\ \hline 20 + 3 \end{array}$ <p>Intermediate step may be needed to lead to clear subtraction understanding.</p> 

<p>Column subtraction with regrouping</p>	 <p>Begin with base 10 or Numicon. Move to pv counters, modelling the exchange of a ten into ten ones.</p>	 <p>Children may draw base ten or PV counters and cross off.</p>	 <p>Begin by partitioning into pv columns</p>  <p>Then move to formal method.</p>
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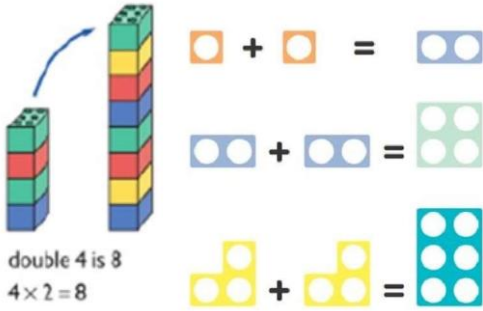

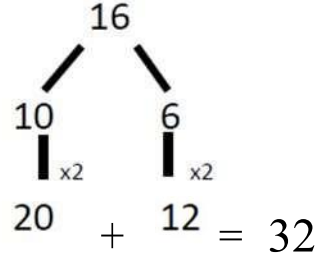
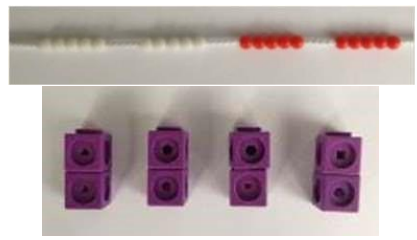
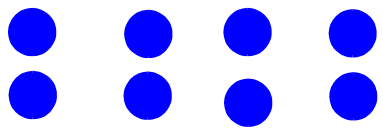
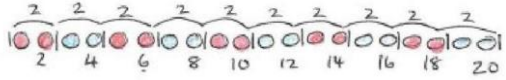
YEARS 4 – 6 SUBTRACTION

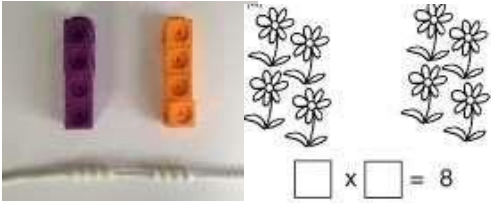

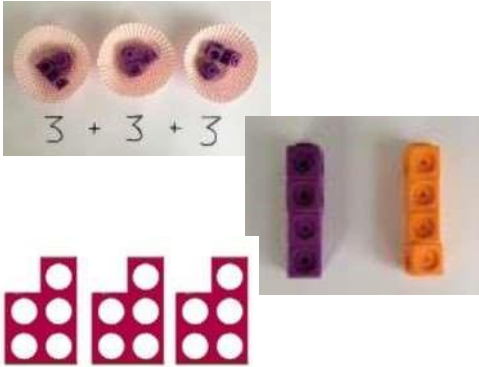
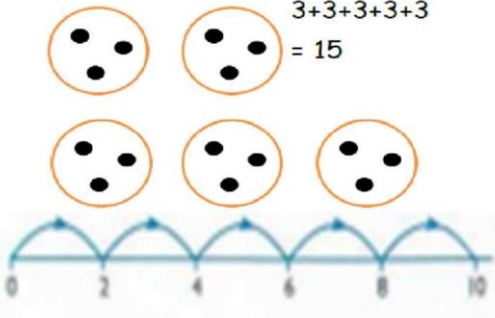

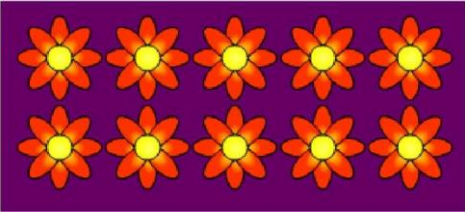
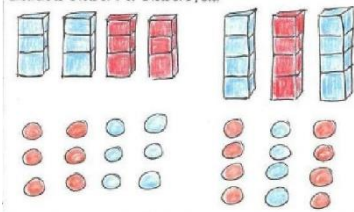
Objective /Strategy	Concrete	Pictorial	Abstract
<p>Subtracting tens and ones</p> <p>Year 4 subtract with up to 4 digits.</p> <p>Introduce decimal subtraction through context of money</p>	<p style="text-align: center;">234 - 179</p>  <p>Model process of exchange using Numicon, base ten and then move to PV counters.</p>	<p>Children to draw pv counters and show their exchange—see Y3</p>	 <p>Use the phrase exchange</p>

<p>Year 5- Subtract with at least 4 digits, including money and measures.</p> <p>Subtract with decimal values, including mixtures of integers and decimals and aligning the decimal Up to 3 decimal places</p>	<p>As Year 4</p>	<p>Children to draw pv counters and show their exchange—see Y3</p>	 <p>Use zeros for placeholder</p>
<p>Year 6—Subtract with increasingly large and more complex numbers and decimal values (up to 3 decimal place).</p>	<p>As Year 4</p>	<p>Children to draw pv counters and show their exchange—see Y3</p>	

YEAR 1 MULTIPLICATION

Programme of Study specifies the following objectives, however it does not require the explicit teaching of the mathematical symbol of multiplication

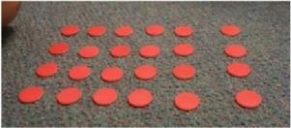



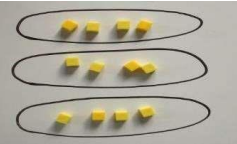
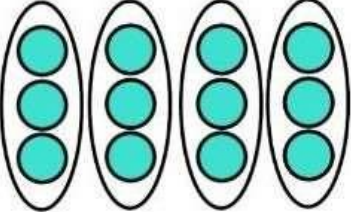
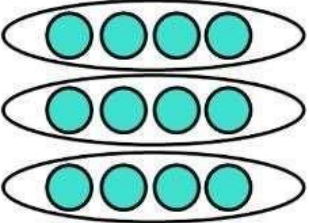

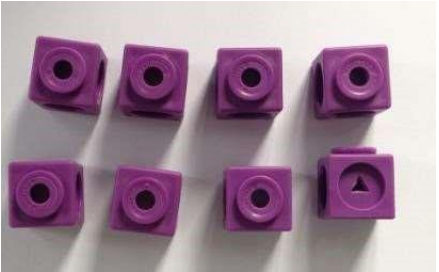
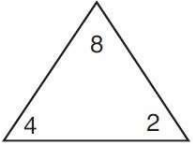
Objective / Strategy	Concrete	Pictorial	Abstract
<p>Doubling</p>	<p>Use practical activities using manipulatives including cubes and Numicon to demonstrate doubling</p>  <p>double 4 is 8 $4 \times 2 = 8$</p>	<p>Draw pictures to show how to double numbers</p> <p>Double 4 is 8</p> 	<p>Partition a number and then double each part before recombining it back together.</p>  <p>$20 + 12 = 32$</p>
<p>Counting in multiples (2s, 5s, 10s)</p>	<p>Count the groups as children are skip counting, children may use their fingers as they are skip counting.</p> 	 <p>Children make representations to show counting in multiples.</p> 	<p>Count in multiples of a number aloud.</p> <p>Write sequences with multiples of numbers.</p> <p>2, 4, 6, 8, 10</p> <p>5, 10, 15, 20, 25, 30</p>

<p>Making equal groups and counting the total.</p>	 <p>Use manipulatives to create equal groups.</p>	<p>Draw  to show $2 \times 3 = 6$</p> <p>Draw and make representations</p>	$2 \times 4 = 8$
<p>Repeated addition</p>	 <p>Use different objects to add equal groups</p>	<p>Use pictorial including number lines to solve prob There are 3 sweets in one bag. How many sweets are in 5 bags altogether?</p>  <p>$3+3+3+3+3 = 15$</p>	<p>Write addition sentences to describe objects and pictures.</p>  <p>$2 + 2 + 2 + 2 + 2 = 10$</p>
<p>Understanding arrays</p>	<p>Use objects laid out in arrays to find the answers to 2 lots 5, 3 lots of 2 etc.</p> 	<p>Draw representations of arrays to show understanding</p> 	$3 \times 2 = 6$ $2 \times 5 = 10$

YEAR 2 MULTIPLICATION

Children should be able to recall and use multiplication and division facts for the 2, 5 and 10 times tables.

Objective / Strategy	Concrete	Pictorial	Abstract
Doubling	<p>Model doubling using dienes and PV</p> <p>counters.</p> $40 + 12 = 52$	<p>Draw pictures and representations to show how to double numbers.</p>	<p>Partition a number and then double each part before recombining it back together.</p> $20 + 12 = 32$
Counting in multiples of 2, 3, 4, 5, 10 from 0 (repeated addition)	<p>Count the groups as children are skip counting, children may use their fingers as they are skip counting. Use bar models.</p> $5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 40$	<p>Number lines, counting sticks and bar models should be used to show representation of counting in multiples.</p>	<p>Count in multiples of a number aloud.</p> <p>Write sequences with multiples of numbers.</p> <p>0, 2, 4, 6, 8, 10</p> <p>0, 3, 6, 9, 12, 15</p> <p>0, 5, 10, 15, 20, 25, 30</p> $4 \times 3 = \square$

Objective / Strategy	Concrete	Pictorial	Abstract
<p>Multiplication is commutative</p>	<p>Create arrays using counters and cubes and Numicon.</p>    <p>Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.</p>  	<p>Use representations of arrays to show different calculations and explore commutativity.</p>  	<p>$12 = 3 \times 4$ $12 = 4 \times 3$</p> <p>Use an array to write multiplication sentences and reinforce repeated addition.</p>  <p>$5 + 5 + 5 = 15$ $3 + 3 + 3 + 3 + 3 = 15$ $5 \times 3 = 15$ $3 \times 5 = 15$</p>
<p>Using the Inverse</p> <p>This should be taught alongside division, so pupils learn how they work alongside each other.</p>		 <p> <input type="text"/> \times <input type="text"/> = <input type="text"/> <input type="text"/> \times <input type="text"/> = <input type="text"/> <input type="text"/> \div <input type="text"/> = <input type="text"/> <input type="text"/> \div <input type="text"/> = <input type="text"/> </p>	<p>$2 \times 4 = 8$ $4 \times 2 = 8$ $8 \div 2 = 4$ $8 \div 4 = 2$ $8 = 2 \times 4$ $8 = 4 \times 2$ $2 = 8 \div 4$ $4 = 8 \div 2$</p> <p>Show all 8 related fact family sentences.</p>

YEAR 3 MULTIPLICATION

Children should be able to recall and use multiplication facts for the 3,4, and 8 times tables

Objective /Strategy

Concrete

Pictorial

Abstract

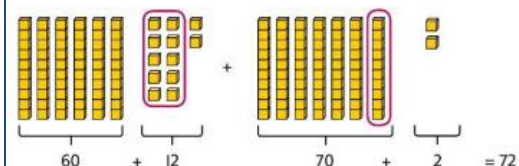
Multiply 2 digit numbers by 1 digit numbers

Use place value equipment to model how 10 ones are exchanged for a 10 in some multiplications.

$$3 \times 24 = ?$$

$$3 \times 20 = 60$$

$$3 \times 4 = 12$$



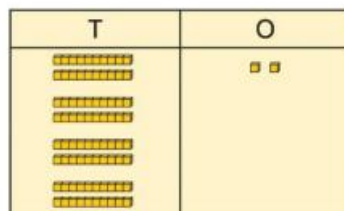
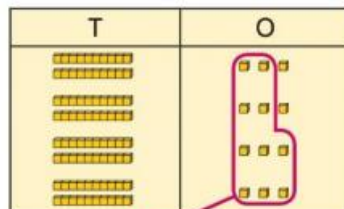
$$3 \times 24 = 60 + 12$$

$$3 \times 24 = 70 + 2$$

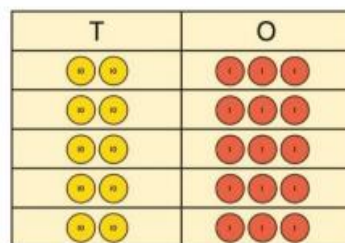
$$3 \times 24 = 72$$

Understand that multiplications may require an exchange of 1s for 10s, and also 10s for 100s.

$$4 \times 23 = ?$$



$$4 \times 23 = 92$$



$$5 \times 23 = ?$$

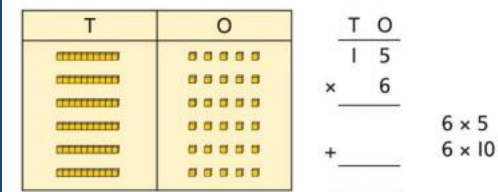
$$5 \times 3 = 15$$

$$5 \times 20 = 100$$

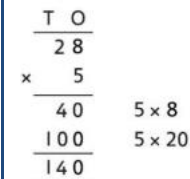
$$5 \times 23 = 115$$

Children may write calculations in expanded column form, but must understand the link with place value and exchange.

Children are encouraged to write the expanded parts of the calculation separately.



$$5 \times 28 = ?$$



Move forward to the formal written method:

$$\begin{array}{r} 35 \\ \times 7 \\ \hline 245 \\ \hline 3 \end{array}$$

Solve problems,
including missing
number problems.

Three times as high, eight times as long

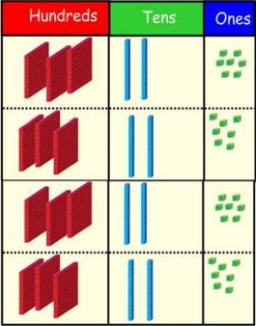
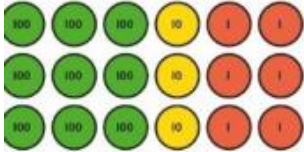
$$? \times 5 = 20$$

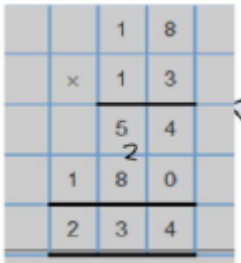
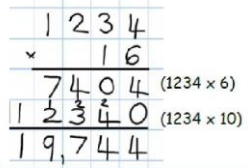
$$20 \div ? = 5$$

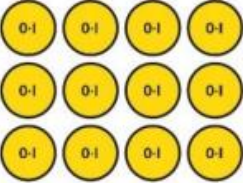
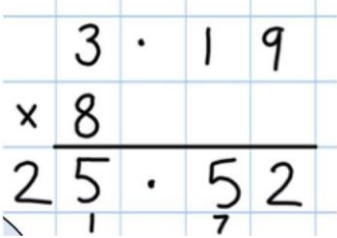
3 hats and 4 coats, how many different
outfits?

YEARS 4 – 6 Multiplication

Objective /Strategy	Concrete	Pictorial	Abstract
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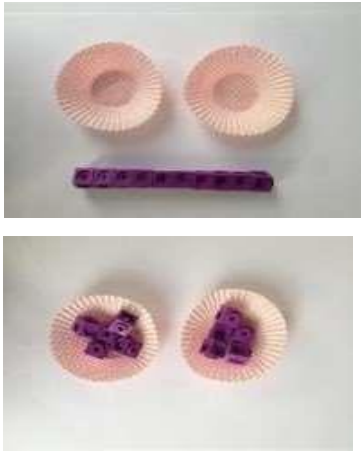
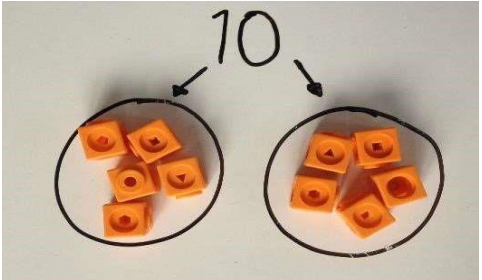

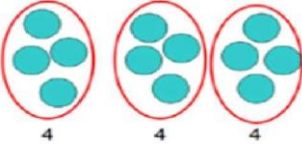
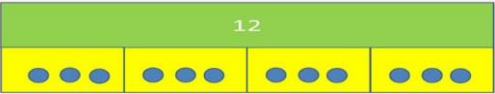
Objective /Strategy	Concrete	Pictorial	Abstract
<p>Column Multiplication for 3 and 4 digits x 1 digit.</p>	 <p>Use place value equipment to make multiplications.</p> <p>It is important at this stage that they always multiply the ones first. Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. $321 \times 2 = 642$</p>	<p>Use place value equipment alongside a column method for multiplication of up to 3-digit numbers by a single digit.</p>  $\begin{array}{r} 321 \\ \times 3 \\ \hline 963 \end{array}$	<p>Use the formal column method for up to 3-digit numbers multiplied by a single digit.</p> $\begin{array}{r} 312 \\ \times 3 \\ \hline 936 \end{array}$ <p>Understand how the expanded column method is related to the formal column method and understand how any exchanges are related to place value at each stage of the calculation.</p> $\begin{array}{r} 23 \\ \times 5 \\ \hline 115 \end{array}$

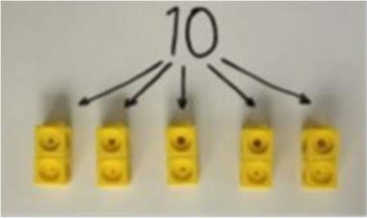
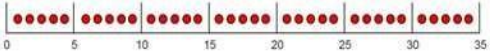
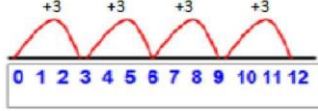
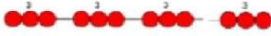

<p>Column multiplication Year 5 & 6</p>	<p>Manipulatives may still be used with the corresponding long multiplication modelled alongside.</p>		<div style="display: flex; align-items: flex-start;"> <div style="flex: 1;">  </div> <div style="flex: 1; padding-left: 10px;"> <p>18 x 3 on the first row (8 x 3 = 24, carrying the 2 for 20, then 1 x 3) 18 x 10 on the 2nd row. Show multiplying by 10 by putting zero in ones column first as a place holder.</p> </div> </div> <div style="margin-top: 20px;">  </div>
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Objective/Strategy	Concrete	Pictorial	Abstract																	
<p>Multiplying decimals up to 2 decimal places by a single digit.</p>	<p>Explore decimal multiplications using place value equipment and in the context of measures.</p> <div style="text-align: center;">  </div> <p><i>3 groups of 4 tenths is 12 tenths. 4 groups of 3 tenths is 12 tenths.</i></p>	<p>Represent calculations of a place value grid.</p> <p>$3 \times 3 = 9$ $3 \times 0.3 = 0.9$</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td style="width: 25px; text-align: center;">T</td> <td style="width: 25px; text-align: center;">O</td> <td style="width: 25px; text-align: center;">•</td> <td style="width: 25px; text-align: center;">Tth</td> </tr> <tr> <td style="height: 40px;"></td> <td></td> <td style="text-align: center;">•</td> <td style="text-align: center;"> <table border="1" style="display: inline-table; border-collapse: collapse;"> <tr><td style="font-size: 8px;">0.1</td><td style="font-size: 8px;">0.1</td><td style="font-size: 8px;">0.1</td></tr> <tr><td style="font-size: 8px;">0.1</td><td style="font-size: 8px;">0.1</td><td style="font-size: 8px;">0.1</td></tr> <tr><td style="font-size: 8px;">0.1</td><td style="font-size: 8px;">0.1</td><td style="font-size: 8px;">0.1</td></tr> </table> </td> </tr> </table>	T	O	•	Tth			•	<table border="1" style="display: inline-table; border-collapse: collapse;"> <tr><td style="font-size: 8px;">0.1</td><td style="font-size: 8px;">0.1</td><td style="font-size: 8px;">0.1</td></tr> <tr><td style="font-size: 8px;">0.1</td><td style="font-size: 8px;">0.1</td><td style="font-size: 8px;">0.1</td></tr> <tr><td style="font-size: 8px;">0.1</td><td style="font-size: 8px;">0.1</td><td style="font-size: 8px;">0.1</td></tr> </table>	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	<p>Remind children that the single digit belongs in the ones column. Line up the decimal points in the question and the answer.</p> <div style="text-align: center; margin-top: 20px;">  </div>
T	O	•	Tth																	
		•	<table border="1" style="display: inline-table; border-collapse: collapse;"> <tr><td style="font-size: 8px;">0.1</td><td style="font-size: 8px;">0.1</td><td style="font-size: 8px;">0.1</td></tr> <tr><td style="font-size: 8px;">0.1</td><td style="font-size: 8px;">0.1</td><td style="font-size: 8px;">0.1</td></tr> <tr><td style="font-size: 8px;">0.1</td><td style="font-size: 8px;">0.1</td><td style="font-size: 8px;">0.1</td></tr> </table>	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1								
0.1	0.1	0.1																		
0.1	0.1	0.1																		
0.1	0.1	0.1																		

YEAR 1 -Division

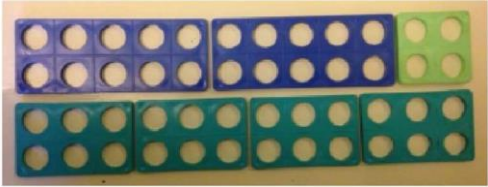
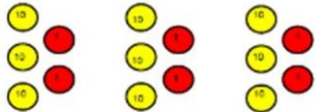
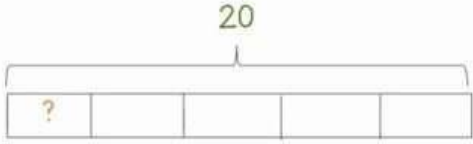
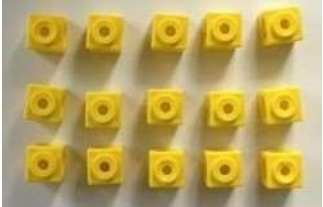
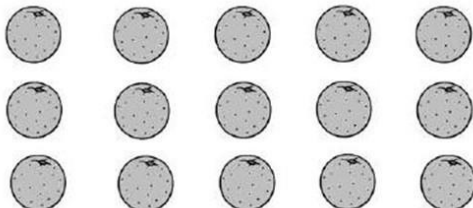
Objective /Strategy	Concrete	Pictorial	Abstract
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Objective/ Strategy	Concrete	Pictorial	Abstract
<p>Division as sharing</p>	 	<p>Children use pictures or shapes to share quantities. .</p>  <p>8 shared between 2 is 4</p> <p>Sharing:</p>  <p style="color: red;">12 shared between 3 is 4</p> 	<p>12 shared between 3 is 4</p>
	<p>I have 10 cubes, can you share them equally in 2 groups?</p>		

Objective/Strategy	Concrete	Pictorial	Abstract
<p>Division as grouping</p>	<p>Divide quantities into equal groups.</p> <p>Use cubes, counters, objects or place value counters to aid understanding.</p>  	<p>Use number lines for grouping</p>   <p>$12 \div 3 = 4$</p> <p>Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.</p>  <p>$20 \div 5 = ?$ $5 \times ? = 20$</p>	<p>$28 \div 7 = 4$</p> <p>Divide 28 into 7 groups. How many are in each group?</p>

YEAR 2 – Division

Objective/Strategy	Concrete	Pictorial	Abstract
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<p>Division as grouping</p>	<p>Use cubes, counters, objects or place value counters to aid understanding.</p>  <p>24 divided into groups of 6 = 4</p> $96 \div 3 = 32$ 	<p>Continue to use bar modelling to aid solving division problems.</p>  $20 \div 5 = ?$ $5 \times ? = 20$	<p>How many groups of 6 in 24?</p> $24 \div 6 = 4$
<p>Division with arrays</p>	 <p>Link division to multiplication by creating an array and thinking about the number sentences that can be created.</p> <p>Eg $15 \div 3 = 5$ $5 \times 3 = 15$ $15 \div 5 = 3$ $3 \times 5 = 15$</p>	<p>Draw an array and use lines to split the array into groups to make multiplication and division sentences</p> 	<p>Find the inverse of multiplication and division sentences by creating eight linking number sentences. $7 \times 4 = 28$</p> $4 \times 7 = 28$ $28 \div 7 = 4$ $28 \div 4 = 7$ $28 = 7 \times 4$ $28 = 4 \times 7$ $4 = 28 \div 7$ $7 = 28 \div 4$

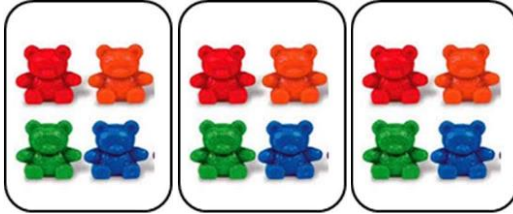
YEAR 3- division

Objective/Strategy	Concrete	Pictorial	Abstract
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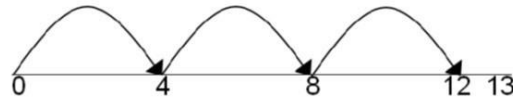
Division with remainders.

$14 \div 3 =$

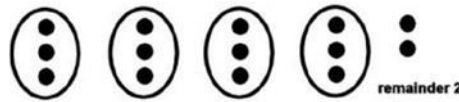
Divide objects between groups and see how much is left over



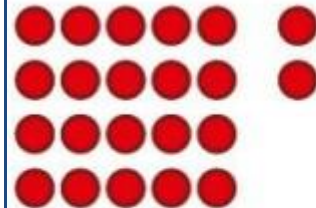
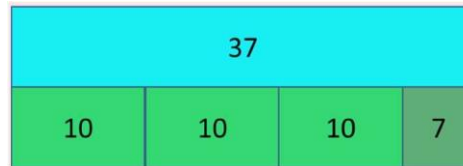
Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.



Draw dots and group them to divide an amount and clearly show a remainder.



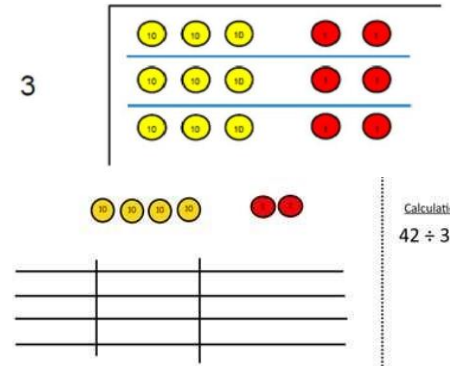
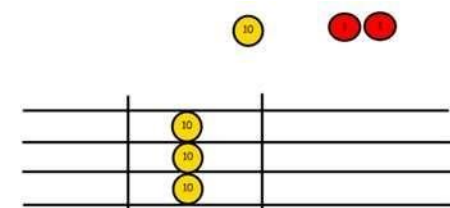
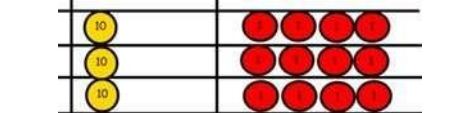
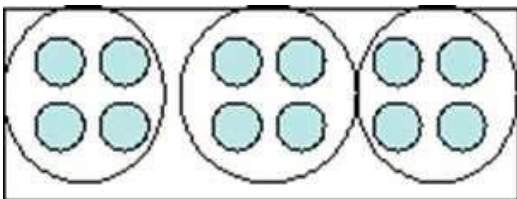
Use bar models to show division with remainders.



$22 \div 5 = 4 \text{ remainder } 2$

Complete written divisions and show the remainder using r.

$$\begin{array}{ccccccc} 29 & \div & 8 & = & 3 & \text{REMAINDER } & 5 \\ \uparrow & & \uparrow & & \uparrow & & \uparrow \\ \text{dividend} & & \text{divisor} & & \text{quotient} & & \text{remainder} \end{array}$$

Objective/Strategy	Concrete	Pictorial	Abstract				
<p>Divide at least 3 digit numbers by 1 digit.</p> <p>Short Division</p>	<p>$96 \div 3$</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td style="text-align: center;">Tens</td> <td style="text-align: center;">Units</td> </tr> <tr> <td style="text-align: center;">3</td> <td style="text-align: center;">2</td> </tr> </table>  <p>Use place value counters to divide using the bus stop method alongside</p> <p>$42 \div 3 =$</p> <p>Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.</p>  <p>We exchange this ten for ten ones and then share the ones equally among the groups.</p>  <p>We look how much in 1 group so the answer is 14.</p>	Tens	Units	3	2	<p>Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.</p>  <p>Encourage them to move towards counting in multiples to divide more efficiently.</p>	<p>Begin with divisions that divide equally with no remainder.</p> $\begin{array}{r} 218 \\ 3 \overline{) 654} \end{array}$ $\begin{array}{r} 4 \\ 4 \overline{) 168} \end{array}$ <p>Move onto divisions with a remainder.</p> $\begin{array}{r} 86 \text{ r } 2 \\ 3 \overline{) 258} \end{array}$ $\begin{array}{r} 0663 \text{ r } 5 \\ 8 \overline{) 5309} \end{array}$ <p>Year 6- divide by two digits using bus stop method. Children to use their preference of long or short division.</p> $\begin{array}{r} 14.6 \\ 35 \overline{) 511.0} \end{array}$
Tens	Units						
3	2						

Y6- Long Division

Step 1—a remainder in the ones

$$\begin{array}{r} \text{h t o} \\ 041\text{R}1 \\ \hline 4 \overline{) 165} \end{array}$$

4 does not go into 1 (hundred). So combine the 1 hundred with the 6 tens (160).

4 goes into 16 four times.

4 goes into 5 once, leaving a remainder of 1.

$$\begin{array}{r} \text{th h t o} \\ 0400\text{R}7 \\ \hline 8 \overline{) 3207} \end{array}$$

8 does not go into 3 of the thousands. So combine the 3 thousands with the 2 hundreds (3,200).

8 goes into 32 four times ($3,200 \div 8 = 400$)

8 goes into 0 zero times (tens).

8 goes into 7 zero times, and leaves a remainder of 7.

Long Division

Step 1 continued...

$$\begin{array}{r} \text{h t o} \\ 061 \\ 4 \overline{) 247} \\ \underline{-4} \\ 3 \end{array}$$

When dividing the ones, 4 goes into 7 one time. Multiply $1 \times 4 = 4$, write that four under the 7, and subtract. This finds us the remainder of 3.

Check: $4 \times 61 + 3 = 247$

$$\begin{array}{r} \text{th h t o} \\ 0402 \\ 4 \overline{) 1609} \\ \underline{-8} \\ 1 \end{array}$$

When dividing the ones, 4 goes into 9 two times. Multiply $2 \times 4 = 8$, write that eight under the 9, and subtract. This finds us the remainder of 1.

Check: $4 \times 402 + 1 = 1,609$

Long Division

Step 2—a remainder in the tens

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
$\begin{array}{r} \text{t o} \\ 2 \overline{) 58} \\ \underline{4} \\ 18 \end{array}$ <p>Two goes into 5 two times, or 5 tens $\div 2 = 2$ whole tens -- but there is a remainder!</p>	$\begin{array}{r} \text{t o} \\ 2 \overline{) 58} \\ \underline{-4} \\ 1 \end{array}$ <p>To find it, multiply $2 \times 2 = 4$, write that 4 under the five, and subtract to find the remainder of 1 ten.</p>	$\begin{array}{r} \text{t o} \\ 29 \\ 2 \overline{) 58} \\ \underline{-4} \\ 18 \end{array}$ <p>Next, drop down the 8 of the ones next to the leftover 1 ten. You combine the remainder ten with 8 ones, and get 18.</p>

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
$\begin{array}{r} \text{t o} \\ 29 \\ 2 \overline{) 58} \\ \underline{-4} \\ 18 \end{array}$ <p>Divide 2 into 18. Place 9 into the quotient.</p>	$\begin{array}{r} \text{t o} \\ 29 \\ 2 \overline{) 58} \\ \underline{-4} \\ 18 \\ \underline{-18} \\ 0 \end{array}$ <p>Multiply $9 \times 2 = 18$, write that 18 under the 18, and subtract.</p>	$\begin{array}{r} \text{t o} \\ 29 \\ 2 \overline{) 58} \\ \underline{-4} \\ 18 \\ \underline{-18} \\ 0 \end{array}$ <p>The division is over since there are no more digits in the dividend. The quotient is 29.</p>

Long Division

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
$\begin{array}{r} \text{h t o} \\ 1 \\ 2 \overline{)278} \\ \underline{-2} \\ 0 \end{array}$ <p>Two goes into 2 one time, or 2 hundreds $\div 2 = 1$ hundred.</p>	$\begin{array}{r} \text{h t o} \\ 1 \\ 2 \overline{)278} \\ \underline{-2} \\ 0 \end{array}$ <p>Multiply $1 \times 2 = 2$, write that 2 under the two, and subtract to find the remainder of zero.</p>	$\begin{array}{r} \text{h t o} \\ 18 \\ 2 \overline{)278} \\ \underline{-2} \downarrow \\ 07 \end{array}$ <p>Next, drop down the 7 of the tens next to the zero.</p>
Divide.	Multiply & subtract.	Drop down the next digit.
$\begin{array}{r} \text{h t o} \\ 13 \\ 2 \overline{)278} \\ \underline{-2} \\ 07 \end{array}$ <p>Divide 2 into 7. Place 3 into the quotient.</p>	$\begin{array}{r} \text{h t o} \\ 13 \\ 2 \overline{)278} \\ \underline{-2} \\ 07 \\ \underline{-6} \\ 1 \end{array}$ <p>Multiply $3 \times 2 = 6$, write that 6 under the 7, and subtract to find the remainder of 1 ten.</p>	$\begin{array}{r} \text{h t o} \\ 13 \\ 2 \overline{)278} \\ \underline{-2} \\ 07 \\ \underline{-6} \\ 18 \end{array}$ <p>Next, drop down the 8 of the ones next to the 1 leftover ten.</p>
1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
$\begin{array}{r} \text{h t o} \\ 139 \\ 2 \overline{)278} \\ \underline{-2} \\ 07 \\ \underline{-6} \\ 18 \end{array}$ <p>Divide 2 into 18. Place 9 into the quotient.</p>	$\begin{array}{r} \text{h t o} \\ 139 \\ 2 \overline{)278} \\ \underline{-2} \\ 07 \\ \underline{-6} \\ 18 \\ \underline{-18} \\ 0 \end{array}$ <p>Multiply $9 \times 2 = 18$, write that 18 under the 18, and subtract to find the remainder of zero.</p>	$\begin{array}{r} \text{h t o} \\ 139 \\ 2 \overline{)278} \\ \underline{-2} \\ 07 \\ \underline{-6} \\ 18 \\ \underline{-18} \\ 0 \end{array}$ <p>There are no more digits to drop down. The quotient is 139.</p>

Step 2—a remainder in any of the place values